Module Description

Vectors and Tensors in Engineering Physics

General Information

Number of ECTS Credits

3

Abbreviation

FTP_Tensors

Version

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Responsible of module

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Language

	Lausanne	Bern	Zürich
Instruction	☐ E ☐ F	☐ D ☐ E ☐ F	☐ D ☒ E
Documentation	☐ E ☐ F	☐ D ☐ E ☐ F	☐ D ☒ E
Examination	☐ E ☐ F	☐ D ☐ E ☐ F	☐ D ☒ E

Module category

☒ Fundamental theoretical principles

☐ Technical/scientific specialization module

☐ Context module

Lessons

☒ 2 lecture periods and 1 tutorial period per week

☐ 2 lecture periods per week

Brief course description of module objectives and content

The course starts with an overview of classical engineering physics with special emphasis of balance and constitutive equations (i.e., continuity equations and material laws). The basic concepts of vector analysis are applied to electrodynamics, various transport phenomena, mechanical elasticity and piezo-electric effects. The concept of tensors enables the description of important anisotropic effects of solid state physics. These effects are present in crystals as well as in layered material systems, which are more and more used in modern technology. The given overview facilitates the student’s understanding and application of numerical simulation methods (e.g., FEA, multiphysics).

Aims, content, methods

Learning objectives and acquired competencies

- Students are familiar with the most important basic laws of engineering physics for isotropic materials in general view form, recognize analogies between different application areas and exploit these for analyzing systems
- Students know about the generalization of the laws for anisotropic materials and can interpret these, especially with regard to application in numerical simulation
- Students master vector analysis and the algebra of tensors together with the standard notation conventions
- Students understand the basics of electrodynamics and transport phenomena for anisotropic systems
- Students understand mechanical elasticity with 3D strain and stress states and are familiar with the material laws in general form for isotropic and anisotropic bodies
- Students understand the piezo-electric effect and its applications in engineering (sensors and actuators)

Contents of module with emphasis on teaching content

- Recapitulation of isotropic material laws (Ohm, Hook, electric polarization, heat conduction)
- Introduction to vector and tensor calculation: scalar, vectorial and tensorial parameters, tensor algebra,
- Transformation behavior of vectors and tensors
- Hands-on calculation of vector analysis and tensoralgebra: electrodynamics and anisotropic transport phenomena
- Elasticity theory with emphasis on 3D stress states
- Piezo-effect: physical fundamentals

Week	Subject
MW1	Introduction, motivation, repetition of fundamental physical laws from engineering physics
MW2	Scalars, vectors, divergence, gradient, curl
MW3	Integral theorems and applications of vector analysis in physics
MW4	Maxwell I: Electro- and magnetostatics
MW5	Maxwell II: Electrodynamics
MW6	Maxwell III: Waves
MW7	Fundamental mathematical properties of tensors, transformations of tensors
MW8	Transport phenomena, Ohm's law, heat conduction and diffusion
MW9	Elasticity: stress and distortion tensor, thermal expansion
MW10	Elasticity: Hooke's law, tensors of the fourth rank, engineering diagram
MW11	Elasticity: 3D stress and distortion states
MW12	Elasticity: 3D stress and distortion states
MW13	Reserve
MW14	Piezoelectricity

Teaching and learning methods

- Frontal teaching (approx. 60 %)
- Presentation and discussion of case studies and problems, individual problem solving (approx. 40 %)

Prerequisites, previous knowledge, entrance competencies

- Physics, analysis, linear algebra at Bachelor's level ,
- The mathematical prerequisites are covered by the chapter 7-9 of [4]. The summaries of these chapters are in the appendix of this document.

Literature

[1] R.E. Newham, Properties of Materials, Oxford, 2005
[2] J.F. Nye, *Physical Properties of Crystals*, Oxford Science Publication, 2004
[3] J.Tichy, *Fundamentals of Piezoelectric Sensorics,* Springer 2010
[4] E. Kreszig, *Advanced Engineering Mathematics*, 10th edition, Wiley, 2011

Assessment

Written module examination

Duration of exam :	120 minutes
Permissible aids:	Personal formula collection, pocket calculator, courseware

Appendix

SUMMARY OF CHAPTER 7

Linear Algebra: Matrices, Vectors, Determinants. Linear Systems

An $m \times n$ **matrix** $\mathbf{A} = [a_{jk}]$ is a rectangular array of numbers or functions ("entries," "elements") arranged in m horizontal **rows** and n vertical **columns**. If $m = n$, the matrix is called **square**. A $1 \times n$ matrix is called a **row vector** and an $m \times 1$ matrix a **column vector** (Sec. 7.1).

The **sum** $\mathbf{A} + \mathbf{B}$ of matrices of the same **size** (i.e., both $m \times n$) is obtained by adding corresponding entries. The **product** of $\mathbf{A}$ by a scalar c is obtained by multiplying each a_{jk} by c (Sec. 7.1).

The **product** $\mathbf{C} = \mathbf{AB}$ of an $m \times n$ matrix $\mathbf{A}$ by an $r \times p$ matrix $\mathbf{B} = [b_{jk}]$ is defined only when $r = n$, and is the $m \times p$ matrix $\mathbf{C} = [c_{jk}]$ with entries

$$c_{jk} = a_{j1}b_{1k} + a_{j2}b_{2k} + \cdots + a_{jn}b_{nk} \qquad \text{(row } j \text{ of } \mathbf{A} \text{ times column } k \text{ of } \mathbf{B}\text{).} \tag{1}$$

This multiplication is motivated by the composition of **linear transformations** (Secs. 7.2, 7.9). It is associative, but is ***not commutative:*** if $\mathbf{AB}$ is defined, $\mathbf{BA}$ may not be defined, but even if $\mathbf{BA}$ is defined, $\mathbf{AB} \neq \mathbf{BA}$ in general. Also $\mathbf{AB} = \mathbf{0}$ may not imply $\mathbf{A} = \mathbf{0}$ or $\mathbf{B} = \mathbf{0}$ or $\mathbf{BA} = \mathbf{0}$ (Secs. 7.2, 7.8). Illustrations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$[1 \quad 2]\begin{bmatrix} 3 \\ 4 \end{bmatrix} = [11], \qquad \begin{bmatrix} 3 \\ 4 \end{bmatrix}[1 \quad 2] = \begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}.$$

The **transpose** $\mathbf{A}^\mathsf{T}$ of a matrix $\mathbf{A} = [a_{jk}]$ is $\mathbf{A}^\mathsf{T} = [a_{kj}]$; rows become columns and conversely (Sec. 7.2). Here, $\mathbf{A}$ need not be square. If it is and $\mathbf{A} = \mathbf{A}^\mathsf{T}$, then $\mathbf{A}$ is called **symmetric;** if $\mathbf{A} = -\mathbf{A}^\mathsf{T}$, it is called **skew-symmetric**. For a product, $(\mathbf{AB})^\mathsf{T} = \mathbf{B}^\mathsf{T}\mathbf{A}^\mathsf{T}$ (Sec. 7.2).

A main application of matrices concerns **linear systems of equations**

$$\mathbf{Ax} = \mathbf{b} \qquad \text{(Sec. 7.3)} \tag{2}$$

(m equations in n unknowns $x_1, \cdots, x_n$; $\mathbf{A}$ and $\mathbf{b}$ given). The most important method of solution is the **Gauss elimination** (Sec. 7.3), which reduces the system to "triangular" form by *elementary row operations,* which leave the set of solutions unchanged. (Numeric aspects and variants, such as *Doolittle's* and *Cholesky's methods,* are discussed in Secs. 20.1 and 20.2.)

SUMMARY OF CHAPTER 8

Linear Algebra: Matrix Eigenvalue Problems

The practical importance of matrix eigenvalue problems can hardly be overrated. The problems are defined by the vector equation

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}. \tag{1}$$

A is a given square matrix. All matrices in this chapter are ***square***. λ is a scalar. To *solve* the problem (1) means to determine values of λ, called **eigenvalues** (or **characteristic values**) of **A**, such that (1) has a nontrivial solution **x** (that is, $\mathbf{x} \neq \mathbf{0}$), called an **eigenvector** of **A** corresponding to that λ. An $n \times n$ matrix has at least one and at most n numerically different eigenvalues. These are the solutions of the **characteristic equation** (Sec. 8.1)

$$D(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{vmatrix} = 0. \tag{2}$$

$D(\lambda)$ is called the **characteristic determinant** of **A**. By expanding it we get the **characteristic polynomial** of **A**, which is of degree n in λ. Some typical applications are shown in Sec. 8.2.

Section 8.3 is devoted to eigenvalue problems for **symmetric** ($\mathbf{A}^\mathsf{T} = \mathbf{A}$), **skew-symmetric** ($\mathbf{A}^\mathsf{T} = -\mathbf{A}$), and **orthogonal matrices** ($\mathbf{A}^\mathsf{T} = \mathbf{A}^{-1}$). Section 8.4 concerns the diagonalization of matrices and the transformation of quadratic forms to principal axes and its relation to eigenvalues.

Section 8.5 extends Sec. 8.3 to the complex analogs of those real matrices, called **Hermitian** ($\mathbf{A}^\mathsf{T} = \mathbf{A}$), **skew-Hermitian** ($\mathbf{A}^\mathsf{T} = -\mathbf{A}$), and **unitary matrices** ($\bar{\mathbf{A}}^\mathsf{T} = \mathbf{A}^{-1}$). All the eigenvalues of a Hermitian matrix (and a symmetric one) are real. For a skew-Hermitian (and a skew-symmetric) matrix they are pure imaginary or zero. For a unitary (and an orthogonal) matrix they have absolute value 1.

The vector product is suggested, for instance, by moments of forces or by rotations. CAUTION! This multiplication is *anti*commutative, $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$, and is *not* associative.

An (oblique) box with edges $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ has volume equal to the absolute value of the **scalar triple product**

$$(\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}) = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}. \tag{7}$$

Sections 9.4–9.9 extend differential calculus to vector functions

$$\mathbf{v}(t) = [v_1(t), v_2(t), v_3(t)] = v_1(t)\mathbf{i} + v_2(t)\mathbf{j} + v_3(t)\mathbf{k}$$

and to vector functions of more than one variable (see below). The derivative of $\mathbf{v}(t)$ is

$$\mathbf{v}' = \frac{d\mathbf{v}}{dt} = \lim_{\Delta t \to 0} \frac{\mathbf{v}(t + \Delta t) - \mathbf{v}(t)}{\Delta t} = [v_1', v_2', v_3'] = v_1'\mathbf{i} + v_2'\mathbf{j} + v_3'\mathbf{k}. \tag{8}$$

Differentiation rules are as in calculus. They imply (Sec. 9.4)

$$(\mathbf{u} \cdot \mathbf{v})' = \mathbf{u}' \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{v}', \qquad (\mathbf{u} \times \mathbf{v})' = \mathbf{u}' \times \mathbf{v} + \mathbf{u} \times \mathbf{v}'.$$

Curves C in space represented by the position vector $\mathbf{r}(t)$ have $\mathbf{r}'(t)$ as a **tangent vector** (the **velocity** in mechanics when t is time), $\mathbf{r}'(s)$ (s arc length, Sec. 9.5) as the *unit tangent vector,* and $|\mathbf{r}''(s)| = \kappa$ as the *curvature* (the *acceleration* in mechanics).

Vector functions $\mathbf{v}(x, y, z) = [v_1(x, y, z), v_2(x, y, z), v_3(x, y, z)]$ represent vector fields in space. Partial derivatives with respect to the Cartesian coordinates x, y, z are obtained componentwise, for instance,

$$\frac{\partial \mathbf{v}}{\partial x} = \left[\frac{\partial v_1}{\partial x}, \frac{\partial v_2}{\partial x}, \frac{\partial v_3}{\partial x}\right] = \frac{\partial v_1}{\partial x}\mathbf{i} + \frac{\partial v_2}{\partial x}\mathbf{j} + \frac{\partial v_3}{\partial x}\mathbf{k} \qquad \text{(Sec. 9.6).}$$

The **gradient** of a scalar function f is

$$\operatorname{grad} f = \nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right] \qquad \text{(Sec. 9.7).} \tag{9}$$

The **directional derivative** of f in the direction of a vector $\mathbf{a}$ is

$$D_{\mathbf{a}} f = \frac{df}{ds} = \frac{1}{|\mathbf{a}|} \mathbf{a} \cdot \nabla f \qquad \text{(Sec. 9.7).} \tag{10}$$

The **divergence** of a vector function $\mathbf{v}$ is

$$\operatorname{div} \mathbf{v} = \nabla \cdot \mathbf{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}. \qquad \text{(Sec. 9.8).} \tag{11}$$

SUMMARY OF CHAPTER 9

Vector Differential Calculus. Grad, Div, Curl

All vectors of the form $\mathbf{a} = [a_1, a_2, a_3] = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ constitute the **real vector space** R^3 with componentwise vector addition

(1) $$[a_1, a_2, a_3] + [b_1, b_2, b_3] = [a_1 + b_1, a_2 + b_2, a_3 + b_3]$$

and componentwise scalar multiplication (c a scalar, a real number)

(2) $$c[a_1, a_2, a_3] = [ca_1, ca_2, ca_3]$$ (Sec. 9.1).

For instance, the *resultant* of forces $\mathbf{a}$ and $\mathbf{b}$ is the sum $\mathbf{a} + \mathbf{b}$.

The **inner product** or **dot product** of two vectors is defined by

(3) $$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \gamma = a_1b_1 + a_2b_2 + a_3b_3$$ (Sec. 9.2)

where γ is the angle between $\mathbf{a}$ and $\mathbf{b}$. This gives for the **norm** or **length** $|\mathbf{a}|$ of $\mathbf{a}$

(4) $$|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}} = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

as well as a formula for γ. If $\mathbf{a} \cdot \mathbf{b} = 0$, we call $\mathbf{a}$ and $\mathbf{b}$ **orthogonal**. The dot product is suggested by the *work* $W = \mathbf{p} \cdot \mathbf{d}$ done by a force $\mathbf{p}$ in a displacement $\mathbf{d}$.

The **vector product** or **cross product** $\mathbf{v} = \mathbf{a} \times \mathbf{b}$ is a vector of length

(5) $$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \gamma$$ (Sec. 9.3)

and perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a}$, $\mathbf{b}$, $\mathbf{v}$ form a *right-handed* triple. In terms of components with respect to right-handed coordinates,

(6) $$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$ (Sec. 9.3).